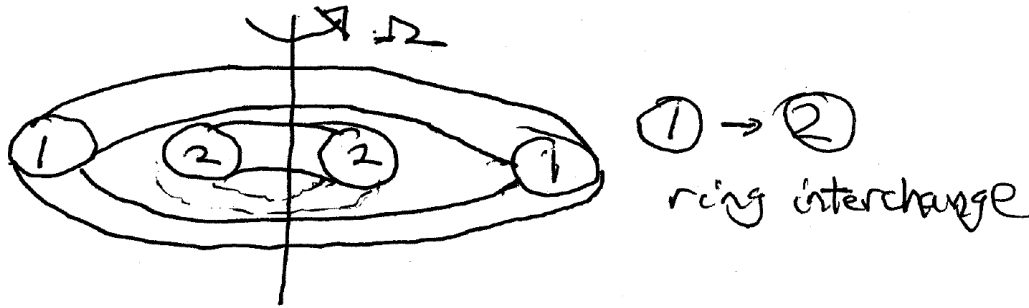


Problem Set III: due TBA

- 1) Kulsrud 5.4
- 2) Kulsrud 7.2
- 3) Consider a fluid in hydrostatic equilibrium with a vertical entropy gradient $\partial S/\partial z < 0$. Take $\underline{g} = -g\hat{z}$.
 - a) Starting from the basic equations, derive the growth rate of ideal Rayleigh-Bernard instability. You will find it helpful to relate the density perturbation $\tilde{\rho}/\rho_0$ to the temperature perturbation \tilde{T}/T_0 by exploiting the fact that the instability develops slowly in comparison to the sound transit time across a cell. Relate your result to the Schwarzschild criterion discussed in class.
 - b) Now, include thermal diffusivity (χ) and viscosity (ν) in your analysis. Calculate the critical temperature gradient for instability, assuming $\chi \sim \nu$. Discuss how this compares to the ideal limit. What happens if $\nu > \chi$?
- 4) Now again, consider the system of Problem 2, now immersed in a uniform magnetic field $\underline{B} = B_0\hat{z}$.
 - a) Assuming ideal dynamics, use the Energy Principle to analyze the stability of a convection cell of vertical wavelength k_z . Of course, $k_z L_p \gg 1$, where L_p is a mean pressure scale length. What is the effect of the magnetic field? Can you estimate how the growth rate changes?
 - b) Now, calculate the growth rate using the full MHD equations. You may assume $\underline{\nabla} \cdot \underline{V} = 0$. What structure convection cell is optimal for vertical transport of heat when B_0 is strong? Explain why. What happens when $B_0 \rightarrow \infty$? Congratulations - you have just derived a variant of the Taylor-Proudman theorem!

- 5) Consider a rotating fluid with mean $\underline{V} = r\Omega(r)\hat{\theta}$. Your task here is to analyze the stability of this system to interchanges of 'rings', i.e.



In all cases, assume $\nabla \cdot \underline{V} = 0$ and $k_\theta = 0$, so the interchange motions carry no angular momentum themselves and the cells sit in the r - z plane.

- 6) Consider magnetic buoyancy interchange instabilities as discussed in class. Assume entropy stratification is neutral, so $dS_0/dz = 0$. Take η small, but non-zero.
- a) Use quasilinear theory to calculate the vertical flux of magnetic intensity. Since, $\Gamma \sim -\partial_z \ln(\langle B \rangle / \rho)$, show that Γ may be written as

$$\Gamma = -D \frac{\partial \langle B \rangle}{\partial z} + V \langle B \rangle.$$

Calculate D, V . Interpret your result. For $\rho = \rho_0(z)$. What profile corresponds to the zero flux state?

- b) What is the origin of the pinch velocity V ? Explain its significance.
- c) As a related example, consider evolution of the particle density according to

$$\partial n / \partial t + \nabla \cdot (n \underline{V}) = 0.$$

Take $n_0 = n_0(x)$, $\underline{B} = B_0(x)\hat{z}$ and $\underline{V} = -\nabla \phi x \hat{z} / B_0(x)$.

Show that density evolution can be related to the incompressible advection of the field n/B :

$$\frac{\partial n}{\partial t} + \underline{V}_{eff} \cdot \nabla (n/B) = 0$$

where $\nabla \cdot \underline{V}_{eff} = 0$.

Show that the mean field equation for $\langle n \rangle$ obeys:

$$\frac{\partial \langle n \rangle}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial}{\partial x} \left(\frac{\langle n \rangle}{\langle B \rangle} \right)$$

where we took $\langle n/B \rangle \cong \langle n \rangle / \langle B \rangle$. Discuss the zero flux state here. What are its implications for the density profile?

Re-write the mean field equation as

$$\frac{\partial \langle n \rangle}{\partial t} = - \frac{\partial}{\partial x} \left[-D \frac{\partial \langle n \rangle}{\partial x} + V \langle n \rangle \right].$$

Relate D and V , here. Under what circumstances will V be inward, i.e. *up* the density gradient?

- d) Relate the results of parts b.), c.) here. What is the lesson?

Congratulations! You have just developed the basics of TEP pinch theory!